Conditional Heteroskedasticity with Risk Spillover Through Networks: An Exponential GARCH Approach

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Outline

Introduction
Motivation

- Networks: geographical, trade, institutional, etc
- Idiosyncratic risk$\Longrightarrow$Network$\Longrightarrow$Systematic Risk

1. Intra-temporal: interactions among traders and policy makers
2. Inter-temporal: reactions on observed historical fluctuations on asset prices

$\Longrightarrow$ need a new model to capture spillover at volatility level to capture both effects

1. Networks and Finance: Kou et al. (2017), Richmond (2019)
2. Conditional Heteroskedasticity: Bollerslev (1990), Engle and Kroner (1995), Engle (2002)
3. Test for Volatility Spillovers between Two Markets: Hong et al. (2001)

Model Formation

Alternative Model Specifications

- Not easy to get a proper extension
- $y_t = \sigma_t \varepsilon_t, \varepsilon_t \overset{i.i.d}{\sim} (0, 1)$
- Extending from linear ARCH/GARCH:

$$\sigma_{i,t}^2 = \mu_i + \lambda \sum_{j=1}^{n} w_{ij,n} y_{j,t}^2 + \gamma y_{i,t-1}^2 + \rho \sum_{j=1}^{n} w_{ij,n} y_{j,t-1}^2$$

- $W_n = (w_{ij,n})_{n \times n}$: spatial correlation matrix among n markets, where $w_{ij,n}$ captures the spillover from market i to market j
- For regularity, we assume $w_{ij,n} \geq 0$ and $w_{ii,n} = 0$ for every $i, j = 1, \cdots, n$

Model Formation

Alternative Model Specifications

- Seems straightforward from SAR, however not a good model:

1. hard to derive moments and other properties
2. hard to be estimated

- Consider the simplest case without inter-temporal terms:

$$y_{i,t}^2 = \mu_i \varepsilon_{i,t}^2 + \lambda \sum_{j=1}^{n} w_{ij,n} y_{j,t}^2 \varepsilon_{i,t}^2$$

Model Formation

Alternative Model Specifications

- Vector Form:

$$\left[I_n - \lambda W_n diag\left(\varepsilon_t^2\right)\right] y_t^2 = diag\left(\mu\right) \varepsilon_t^2$$

- Two situations:

1. ε continuous on $\mathbb{R} \Longrightarrow \left[I_n - \lambda W_n diag\left(\varepsilon_t^2\right)\right]^{-1}$ can not be simplified
2. ε with bounded support:

$$y_t^2 = \sum_{l=0}^{\infty} \lambda^l \left[W_n diag\left(\varepsilon_t^2\right)\right]^l diag\left(\mu\right) \varepsilon_t^2$$

$\Longrightarrow$ extremely hard to derive moments, also hard to establish bijection projection

Model Formation

DGP of ESPARCH(1,1) Model

- Extension from EGARCH and focus on dynamic of conditional log-volatility:

$$\ln\left(\sigma_t^2\right) = \alpha_t + \sum_{k=1}^{\infty} \beta_k g\left(y_{i,t-k}\right) + \sum_{s=0}^{\infty} \sum_{j=1, j\neq i}^{n} \lambda_k w_{ij,n} g\left(y_{j,t-s}\right)$$

- where $g(x) = \ln x^2$ for $x \neq 0$
- $\lambda_k w_{ij,n} g\left(y_{j,t-k}\right)$ captures the inter-temporal spillover effect from i to j on conditional volatility $s > 0$ periods ago
- When $s = 0$, it captures the intra-temporal spillover effect

Model Formation

DGP of ESPARCH(1,1) Model

- ESPARCH(1,1):

$$y_{i,t} = \sigma_{i,t}\varepsilon_{i,t}, \varepsilon_{i,t} \overset{iid}{\sim}(0,1)$$

$$\ln \sigma_{i,t}^2 = \mu_i + \lambda \sum_{j=1}^{n} w_{ij,n} \ln y_{j,t}^2 + \gamma \ln y_{i,t-1}^2 + \rho \sum_{j=1}^{n} w_{ij,n} \ln y_{j,t-1}^2$$

- The order of spatial lag and time lag are both 1, i.e. we only consider risk-spillover through one particular network and only consider dynamic effect from the previous period

Model Formation

DGP of ESPARCH(1,1) Model

- Economic Meaning

$$\lambda w_{ij,n} = \frac{\partial \ln \sigma_{i,t}^2}{\partial \ln y_{j,t}^2} \approx \frac{\triangle \sigma_{i,t}^2 / \sigma_{i,t}^2}{\triangle y_{j,t}^2 / y_{j,t}^2}$$

$$\gamma = \frac{\partial \ln \sigma_{i,t}^2}{\partial \ln y_{i,t-1}^2} \approx \frac{\triangle \sigma_{i,t}^2 / \sigma_{i,t}^2}{\triangle y_{i,t-1}^2 / y_{i,t-1}^2}$$

$$\rho w_{ij,n} = \frac{\partial \ln \sigma_{i,t}^2}{\partial \ln y_{j,t-1}^2} \approx \frac{\triangle \sigma_{i,t}^2 / \sigma_{i,t}^2}{\triangle y_{j,t-1}^2 / y_{j,t-1}^2}$$

- λ, γ and ρ capture the elasticity of conditional volatility with respect to the volatility of other assets and historical volatility of its own and other assets

Model Formation

Covariance Stationarity of ln $y_{i,t}^2$

- VAR form for any fixed n:

$$
\begin{aligned}
logY_t^2 &= (I_n - \lambda W_n)^{-1}(\gamma I_n + \rho W_n)\, logY_{t-1}^2 \\
&\quad + (I_n - \lambda W_n)^{-1}(\mu + \omega) + (I_n - \lambda W_n)^{-1}\xi_t
\end{aligned}
$$

- $E\left(log\varepsilon_t^2\right) = \omega$ and $\xi_t = log\varepsilon_t^2 - \omega$, $(I_n - \rho W_n)^{-1}$ exists
- Necessary condition for stationarity: $\left\|(I_n - \lambda W_n)^{-1}(\gamma I_n + \rho W_n)\right\|_\infty < 1$
- When W_n is row-normalized, i.e. $\sum_{j=1}^n w_{ij} = 1$ for $\forall i$, we need $|\lambda| + |\gamma| + |\rho| < 1$

QMLE and Asymptotic Properties

QMLE for Normal Disturbance

- $\varepsilon_{i,t} \overset{i.i.d}{\sim} N(0,1) \Longrightarrow E\left(\ln \varepsilon_{i,t}^2\right) = \psi\left(\frac{1}{2}\right) - \ln\left(\frac{1}{2}\right) \approx -1.27$, $Var\left(\ln \varepsilon_{i,t}^2\right) = \frac{1}{2}\pi^2$
- Let $logY_t^2 = Z_t$ and $\eta = \mu - 1.27 l_n$ where $l_n = \left(\underbrace{1, \cdots 1}_{n}\right)'$, we have

$$Z_t = \eta + \lambda W_n Z_t + (\gamma I_n + \rho W_n) Z_{t-1} + \xi_t$$

- Based on this linearized model, we can use QMLE method using Normal density as approximation

- Assume normality of ξ_t, then the conditional quasi-log-density function for $t = 1, \cdots, T$ is

$$
\begin{aligned}
& q_{n,t}\left(X_t; \theta, \eta | \mathscr{F}_{t-1}\right) \\
= & -\frac{3}{2} n \ln(\pi) - \frac{1}{\pi^2}\left[S_n(\lambda) Z_t - (\gamma I_n + \rho W_n) Z_{t-1} - \eta\right]' \\
& \cdot \left[S_n(\lambda) Z_t - (\gamma I_n + \rho W_n) Z_{t-1} - \eta\right] + \ln\left|S_n(\lambda)\right|
\end{aligned}
$$

- Then, quasi-log-likelihood function is

$$
\begin{aligned}
& Q_{n,T}(\theta, \eta) \\
= & -\frac{3}{2} n T \ln(\pi) - \frac{1}{\pi^2} \sum_{t=1}^{T}\left[S_n(\lambda) Z_t - (\gamma I_n + \rho W_n) Z_{t-1} - \eta\right]' \\
& \cdot \left[S_n(\lambda) Z_t - (\gamma I_n + \rho W_n) Z_{t-1} - \eta\right] + T \ln\left|S_n(\lambda)\right|
\end{aligned}
$$

- FOC of η: $-\frac{2}{\pi^2}\sum_{t=1}^{T}\left[S_n(\lambda)Z_t-(\gamma I_n+\rho W_n)Z_{t-1}-\eta\right]'=0$
- Concentrated QMLE:

$$\begin{aligned}&\widetilde{Q}_{n,T}(\theta)\\&=-\frac{3}{2}nT\ln(\pi)-\frac{1}{\pi^2}\sum_{t=1}^{T}\left[S_n(\lambda)\widetilde{Z}_t-(\gamma I_n+\rho W_n)\widetilde{Z}_{t-1}\right]'\\&\cdot\left[S_n(\lambda)\widetilde{Z}_t-(\gamma I_n+\rho W_n)\widetilde{Z}_{t-1}\right]+T\ln\left|S_n(\lambda)\right|\end{aligned}$$

- $\widetilde{Z}_t=Z_t-\frac{1}{T}\sum_{t=1}^{T}Z_t$
- $\hat{\eta}=\frac{1}{T}\sum_{t=1}^{T}\left[S_n\left(\hat{\lambda}\right)Z_t-(\hat{\gamma}I_n+\hat{\rho}W_n)Z_{t-1}\right]$

QMLE and Asymptotic Properties

QMLE for non-Normal Disturbance: t-distribution

- t-distribution:$\varepsilon_{i,t} \overset{i.i.d}{\sim} \sqrt{\frac{v-2}{v}} t\left(v\right)$ for $v \geq 3$, we can rewrite it as $\varepsilon_{i,t} = \sqrt{\frac{v-2}{v}} \zeta_{i,t} / \kappa_{i,t}^{\frac{1}{2}}$
- $\ln \varepsilon_{i,t}^2 = \ln\left(\frac{v-2}{v}\right) + \ln \zeta_{i,t}^2 - \ln \kappa_{i,t}$ where $\zeta_{i,t} \overset{i.i.d}{\sim} N\left(0,1\right)$ and $\kappa_{i,t} \overset{i.i.d}{\sim} \chi^2\left(v\right)$ with degree of freedom v
- $E\left(\ln \varepsilon_{i,t}^2\right) = \ln\left(\frac{v-2}{v}\right) - \psi\left(\frac{v}{2}\right) + \psi\left(\frac{1}{2}\right)$
- $var\left(\ln \varepsilon_{i,t}^2\right) = \frac{1}{2}\pi^2 + \psi'\left(\frac{v}{2}\right)$
- By similar process as Normal situation, we can estimate the parameters by QMLE

QMLE and Asymptotic Properties

QMLE for non-Normal Disturbance: unknown distribution

- distribution of $\varepsilon_{i,t}$ unknown: similar way
- concentrated QMLE: with $var\left(\ln \varepsilon_{i,n}^2\right) = \sigma^2$, we have

$$
\begin{aligned}
& \widetilde{H}_{n,T}\left(\sigma^2, \theta\right) \\
&= -\frac{nT}{2} \ln\left(2\pi\sigma^2\right) - \frac{1}{2\sigma^2} \sum_{t=1}^{T} \left[S_n(\lambda)\widetilde{Z}_t - \left(\gamma I_n + \rho W_n\right)\widetilde{Z}_{t-1}\right]' \\
&\cdot \left[S_n(\lambda)\widetilde{Z}_t - \left(\gamma I_n + \rho W_n\right)\widetilde{Z}_{t-1}\right] + T \ln\left|S_n(\lambda)\right|
\end{aligned}
$$

- limitation: fixed effect μ can not be identified since $E\left(\ln \varepsilon_{i,n}^2\right)$ is unknown

QMLE and Asymptotic Properties

Asymptotic Properties

- Based on Yu et al. (2008), when $n/T \to 0$, for $\psi = \left(\sigma^2, \theta'\right)'$, we have

$$\sqrt{n}\left(\hat{\psi}_{nT} - \psi_0\right) \xrightarrow{d} N\left(0, \Sigma_{\psi_0}^{-1}\left(\Sigma_{\psi_0} + \Omega_{\psi_0}\right)\Sigma_{\psi_0}^{-1}\right)$$

- $\Sigma_{\psi_0} = E\left(\frac{1}{nT}\frac{\partial^2 \widetilde{Q}_{n,T}(\psi_0)}{\partial\psi'\partial\psi}\right)$
- $\Omega_{\psi_0} = \left(\frac{4\mu_4}{\sigma_0^4} - 3\right)\begin{pmatrix} \frac{1}{4\sigma_0^4} & \frac{1}{2\sigma_0^2 n}tr(G_n) & \\ \frac{1}{2\sigma_0^2 n}tr(G_n) & \frac{1}{n}\sum_{i=1}^{n} G_{n,ii}^2 & 0_{2\times 2} \\ 0_{2\times 2} & & 0_{2\times 2} \end{pmatrix}$ for $E\left|\xi_{i,t}\right|^{4+\epsilon} < \infty$
- For normal and t-distribution scenario, no σ^2 terms (restricted model)

QMLE and Asymptotic Properties

Asymptotic Properties

- Fixed effect η: for $i = 1, \cdots, n$

$$\sqrt{T}\left(\hat{\eta}_{i,nT} - \eta_{i,0}\right) \xrightarrow{d} N\left(0, \sigma_0^2\right)$$

- asymptotically independent with each other
- For $n/T \to \infty$ situation, still consistent but asymptotic distribution is not symmetric and depend on the the distribution of ξ

QMLE and Asymptotic Properties

Test for Normality

- Want to test whether $\varepsilon_{i,t}$ is normal $\Rightarrow$ forecast and construct confidence interval
- Similar to stochastic volatility models, due to log-transformation, no way to directly test normality
- Ruiz (1994): test based on moment of $\ln \varepsilon_{i,t}^2$, i.e. $H_0 : \sigma^2 = \frac{1}{2}\pi^2$ v.s. $H_1 : \sigma^2 \neq \frac{1}{2}\pi^2$
- FOC of σ^2 for unrestricted model:

$$g_{\sigma^2}\left(\bar{\psi}\right) = -\frac{nT}{\pi^2} + \frac{2}{\pi^4}\sum_{t=1}^{T} \bar{u}_{c,t}^{\prime}\bar{u}_{c,t}$$

QMLE and Asymptotic Properties

Test for Normality

- Modified LM statistic: for $g_{n,T}\left(\bar{\psi}\right) = \left(g_{\sigma^2}\left(\bar{\psi}\right) + \frac{n}{\pi^2}, 0, 0, 0\right)'$

$$
H_{n,T}\left(\bar{\psi}\right) = T\left(\frac{4\mu_4}{\pi^4} - 3\right)\left(\begin{array}{cc} \begin{array}{cc} \frac{n}{\pi^4} & \frac{1}{\pi^2}tr(G_n) \\ \frac{1}{\pi^2}tr(G_n) & \sum_{i=1}^{n} G_{n,ii}^2 \end{array} & 0_{2\times 2} \\ 0_{2\times 2} & 0_{2\times 2} \end{array}\right) - E\left(\frac{\partial^2 \widetilde{H}_{n,T}\left(\bar{\psi}\right)}{\partial\psi'\partial\psi}\right)
$$

$$
\Longrightarrow J_{Norm} = g_{n,T}'\left(\bar{\psi}\right) H_{n,T}^{-1}\left(\bar{\psi}\right) g_{n,T}'\left(\bar{\psi}\right) \xrightarrow{d} \chi^2\left(1\right)
$$

QMLE and Asymptotic Properties

Test for Normality

- Limitation of the J_{Norm} statistic: only use second order and third order moment
- For some particular distribution, $\ln \varepsilon^2_{i,t}$ can be close to log-χ^2

  e.g. extreme value distribution

- Not a huge problem: not affect inference on parameters of risk-spillovers

Monte Carlo Simulations

Basic Settings

- Simulation network adjacent weighting matrix:

1. Generate two random vectors of coordinates as the geographic location for each observation;
2. Find l nearest neighbors for each observation according to their spatial distances and denote the corresponding $w_{ij,n} = 1$, otherwise $w_{ij,n} = 0$;
3. Row-normalize W_n

- We consider two different situations when $l = 3$ and $l = 6$

Monte Carlo Simulations

Basic Settings

- Fixed effect μ_i: random draw from $i.i.d$ uniform distribution on $[0, 1]$
- Parameters: $(\lambda_1, \gamma_1, \rho_1) = (.4, .2, -.3)$ and $(\lambda_2, \gamma_2, \rho_2) = (-.3, .4, .2)$
- We replicate each Monte Carlo simulation exercise by 1,000 times
- True value of σ^2 depends on the distribution of $\varepsilon_{i,n}$

Monte Carlo Simulations

Normal Situation

Table 1: Finite Sample Performance for $N(0,1)$ Case When $l = 3$

		$(\lambda_1, \gamma_1, \rho_1) = (.4, .2, -.3)$			$(\lambda_2, \gamma_2, \rho_2) = (-.3, .4, .2)$		
		λ	γ	ρ	λ	γ	ρ
	mean	.3859	.1626	-.2743	-.2947	.3467	.1763
n=10	std	.0550	.0560	.0800	.0782	.0550	.1012
	med	.3870	.1615	-.2765	-.2941	.3476	.1790
T=30	$q_{0.25}$	.3473	.1260	-.3300	-.3452	.3091	.1050
	$q_{0.75}$	.4254	.1997	-.2225	.2431	.3865	.2466
	mean	.3964	.1903	-.2923	-.3001	.3849	.1921
n=10	std	.0315	.0295	.0416	.0422	.0291	.0574
	med	.3972	.1906	-.2923	-.2993	.3856	.1924
T=100	$q_{0.25}$	.3751	.1711	-.3205	-.3282	.3648	.1516
	$q_{0.75}$	.4175	.2106	-.2641	-.2709	.4038	.2313
	mean	.3980	.1881	-.2933	-.2979	.3858	.1932
n=30	std	.0200	.0170	.0242	.0266	.0172	.0337
	med	.3988	.1881	-.2933	-.2988	.3859	.1933
T=100	$q_{0.25}$	.3852	.1768	-.3097	-.3157	.3745	.1696
	$q_{0.75}$	.4120	.2002	-.2778	-.2799	.3971	.2160

Monte Carlo Simulations

$t-$distribution Situation

Table 4: Finite Sample Performance for $\sqrt{\frac{1}{3}}t\,(3)$ Case When $l = 6$

		$(\lambda_1, \gamma_1, \rho_1) = (.4, .2, -.3)$			$(\lambda_2, \gamma_2, \rho_2) = (-.3, .4, .2)$		
		λ	γ	ρ	λ	γ	ρ
	mean	.3808	.1590	-.2721	-.3030	.3449	.1622
n=10	std	.0725	.0599	.1025	.1231	.0564	.1644
	med	.3883	.1593	-.2710	-.2954	.3452	.1676
T=30	$q_{0.25}$	.3359	.1176	-.3418	-.3811	.3075	.0650
	$q_{0.75}$	.4302	.2007	-.2015	-.2188	.3826	.2643
	mean	.3943	.1876	-.2939	-.3053	.3850	.1949
n=10	std	.0374	.0310	.0542	.0659	.0297	.0876
	med	.3970	.1870	-.2923	-.3028	.3841	.1978
T=100	$q_{0.25}$	.3701	.1674	-.3303	-.3463	.3649	.1343
	$q_{0.75}$	.4201	.2092	-.2589	-.2618	.4073	.2513
	mean	.3978	.1889	-.2935	-.2977	.3853	.1935
n=30	std	.0259	.0181	.0344	.0382	.0177	.0489
	med	.3977	.1888	-.2926	-.2965	.3854	.1946
T=100	$q_{0.25}$	.3809	.1766	-.3171	-.3221	.3730	.1615
	$q_{0.75}$	.4151	.2016	-.2698	-.2721	.3981	.2250

Monte Carlo Simulations

Unknown Distribution

- We consider the following three distributions:

1. $\varepsilon_{i,t} \overset{iid}{\sim} \frac{1}{\sqrt{3}} t\,(3)$
2. Subrahmanyam (1994) and Kim and Rhee (1998): $\varepsilon_{i,t} \overset{iid}{\sim} \mathit{Uniform}\left[-\sqrt{3}, \sqrt{3}\right]$
3. Harvey and Siddique (2000) and Chang, Christofferen and Jacobs (2013): Extreme value distribution

$$f\,(x) = \exp\left\{\frac{\pi}{\sqrt{6}}\,(x - \gamma_{EM}) - \exp\left\{\frac{\pi}{\sqrt{6}}\,(x - \gamma_{EM})\right\}\right\}, x \in \mathbb{R}$$

Monte Carlo Simulations

Unknown Distribution

Table 5: Finite Sample Performance for Unknown $\sqrt{\frac{1}{3}}t\,(3)$ Case When $l = 3$

		$\sigma^2 \approx 5.87$							
		$(\lambda_1, \gamma_1, \rho_1) = (.4, .2, -.3)$				$(\lambda_2, \gamma_2, \rho_2) = (-.3, .4, .2)$			
		λ	γ	ρ	σ^2	λ	γ	ρ	σ^2
	mean	.3924	.1562	-.2755	5.6420	-.3064	.3480	.1891	5.5664
n=10	std	.0605	.0574	.0812	.7220	.0902	.0542	.1136	.7160
	med	.3976	.1571	-.2782	5.5892	-.3031	.3485	.1872	5.5077
T=30	$q_{0.25}$	.3533	.1167	-.3311	5.1516	-.3640	.3126	.1097	5.0692
	$q_{0.75}$	.4335	.1978	-.2202	6.0903	-.2468	.3867	.2630	5.9948
	mean	.3969	.1886	-.2922	5.8217	-.2879	.3839	.1950	5.7795
n=10	std	.0314	.0317	.0419	.4168	.0462	.0292	.0581	.3947
	med	.3965	.1877	-.2925	5.7900	-.2980	.3849	.1960	5.7729
T=100	$q_{0.25}$	.3766	.1684	-.3192	5.5310	-.3291	.3634	.1561	5.5018
	$q_{0.75}$	.4183	.2109	-.2651	6.0963	-.2653	.4044	.2330	6.0410
	mean	.3969	.1882	-.2943	5.7962	-.3002	.3852	.1946	5.7881
n=30	std	.0197	.0175	.0264	.0243	.0259	.0170	.0324	.2296
	med	.4003	.1880	-.2950	5.7873	-.3008	.3851	.1954	5.7925
T=100	$q_{0.25}$	.3866	.1763	-.3135	5.6314	-.3167	.3738	.1724	5.6381
	$q_{0.75}$	.4131	.1999	-.2760	5.9518	-.2827	.3960	.2165	5.9366

Monte Carlo Simulations

Unknown Distribution

Table 8: Finite Sample Performance for Unknown $U\left[-\sqrt{3},\sqrt{3}\right]$ Case When $l=6$

		$\sigma^2=4$							
		$(\lambda_1,\gamma_1,\rho_1)=(.4,.2,-.3)$				$(\lambda_2,\gamma_2,\rho_2)=(-.3,.4,.2)$			
		λ	γ	ρ	σ^2	λ	γ	ρ	σ^2
	mean	.3814	.1594	-.2790	3.8574	-.3117	.3484	.1760	3.8180
n=10	std	.0719	.0556	.1034	.6281	.1183	.0524	.1597	.6444
	med	.3899	.1587	-.2808	3.8148	-.3060	.3480	.1773	3.7475
T=30	$q_{0.25}$	.3335	.1205	-.3533	3.4135	-.3874	.3112	.0661	3.3790
	$q_{0.75}$	.4346	.1969	-.2060	4.2552	-.2295	.3841	.2860	4.2158
	mean	.3959	.1878	-.2924	3.9546	-.3038	.3834	.1931	3.9595
n=10	std	.0411	.0313	.0578	.3677	.0737	.0298	.0915	.3587
	med	.3971	.1883	-.2918	3.9139	-.3046	.3843	.1929	3.9338
T=100	$q_{0.25}$	.3698	.1659	-.3312	3.6984	-.3504	.3644	.1381	3.7223
	$q_{0.75}$	.4230	.2086	-.2542	4.1892	-.2538	.4043	.2567	4.1855
	mean	.3978	.1868	-.2939	3.9517	-.3025	.3850	.1935	3.9478
n=30	std	.0283	.0176	.0342	.2062	.0387	.0169	.0543	.2057
	med	.3991	.1868	-.2936	3.9506	-.3024	.3852	.1950	3.9508
T=100	$q_{0.25}$	.3803	.1753	-.3154	3.8131	-.3260	.3737	.1634	3.8078
	$q_{0.75}$	.4156	.1984	-.2721	4.0904	-.2768	.3968	.2302	4.0800

Monte Carlo Simulations

Unknown Distribution

Table 10: Finite Sample Performance for Unknown EV Case When $l = 6$

		$\sigma^2 \approx 4.89$							
		$(\lambda_1, \gamma_1, \rho_1) = (.4, .2, -.3)$				$(\lambda_2, \gamma_2, \rho_2) = (-.3, .4, .2)$			
		λ	γ	ρ	σ^2	λ	γ	ρ	σ^2
	mean	.3816	.1588	-.2765	4.6974	-.3134	.3463	.1677	4.6793
n=10	std	.0717	.0578	.1087	.6845	.1315	.0551	.1694	.6756
	med	.3845	.1597	-.2740	4.6340	-.3082	.3481	.1720	4.6314
T=30	$q_{0.25}$	.3369	.1181	-.3508	4.2146	-.3946	.3090	.0579	4.1874
	$q_{0.75}$	.4321	.1990	-.2060	5.1251	-.2251	.3846	.2799	5.1076
	mean	.3944	.1876	-.2913	4.8291	-.3080	.3840	.1885	4.8254
n=10	std	.0403	.0313	.0560	.3919	.0716	.0291	.0982	.3744
	med	.3950	.1858	-.2907	4.8117	-.3034	.3844	.1929	4.8080
T=100	$q_{0.25}$	.3676	.1662	-.3265	4.5680	-.3542	.3653	.1255	4.5538
	$q_{0.75}$	.4229	.2076	-.2560	5.0849	-.2590	.4021	.2552	5.0638
	mean	.3978	.1877	-.2944	4.8379	-.3030	.3851	.1919	4.8369
n=30	std	.0245	.0176	.0341	.2082	.0395	.0170	.0577	.2044
	med	.3984	.1872	-.2944	4.8317	-.3032	.3853	.1958	4.8380
T=100	$q_{0.25}$	.3810	.1759	-.3168	4.6916	-.3305	.3740	.1627	4.6806
	$q_{0.75}$	.4154	.1994	-.2725	4.9859	-.2773	.3970	.2266	4.9771

Monte Carlo Simulations

Performance of Normality Test

- Test size: as T goes larger, approaching to theoretical value, over reject when sample size is small

Table 11: Test Size of J_{Norm} ($\chi^2_{0.95}(1) = 3.8415$)

		$(\lambda_1, \gamma_1, \rho_1) = (.4, .2, -.3)$		$(\lambda_2, \gamma_2, \rho_2) = (-.3, .4, .2)$	
n	T	$l = 3$	$l = 6$	$l = 3$	$l = 6$
10	30	.113	.073	.065	.048
	100	.099	.069	.059	.045
	200	.092	.058	.054	.052
30	30	.084	.043	.064	.041
	100	.092	.065	.061	.052
	200	.077	.061	.057	.051

Monte Carlo Simulations

Performance of Normality Test

- Test Power: serious lack of power issue when true distribution of $\varepsilon_{i,t}$ follows extreme value distribution, work well for other scenarios

Table 13: Test Power of J_{Norm} When $l = 6$ $(\chi^2_{0.95}(1) = 3.8415)$

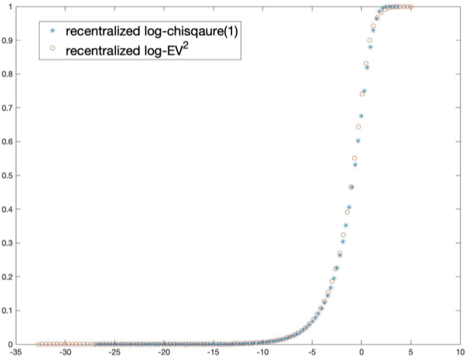
		$(\lambda_1, \gamma_1, \rho_1) = (.4, .2, -.3)$			$(\lambda_2, \gamma_2, \rho_2) = (-.3, .4, .2)$		
n	T	$\sqrt{\frac{1}{3}}t(3)$	$U\left[-\sqrt{3}, \sqrt{3}\right]$	EV	$\sqrt{\frac{1}{3}}t(3)$	$U\left[-\sqrt{3}, \sqrt{3}\right]$	EV
10	30	.213	.399	.058	.185	.367	.063
	100	.645	.800	.064	.605	.754	.062
	200	.926	.959	.058	.911	.959	.049
30	30	.588	.747	.064	.540	.723	.055
	100	.992	.991	.065	.991	.994	.066
	200	1	1	.068	1	1	.065

Monte Carlo Simulations

Performance of Normality Test

- Problem: distributions of $\xi_{i,t}$ are too similar

Figure 2: Comparison of Kernel CDF of 100,000 Random Sample of Re-centralized $\ln \varepsilon_{i,t}^2$



Monte Carlo Simulations

Performance of Normality Test

- Non-parametric Kolmogorov-Smirnov test also hard to distinguish these two distribution with small samples

Table 16: Simulated Test Power of Two Sample K-S Test For Re-centralized $\ln \varepsilon_{i,t}^2$

	$\alpha = .1$	$\alpha = .05$	$\alpha = .01$
$n = 1000$	.1394	.0751	.0169
$n = 5000$	.5477	.4196	.2026
$n = 10000$	.9013	.7926	.5566

- Since second and third moments are very close, does not affect statistical inference

Application: Risk Spillover Among Eurozone Countries
Data Description

- 11 original eurozone countries: Belgium, Germany, Spain, France, Italy, Netherland, Portugal, Austria, Finland, Luxembourg, Ireland
- monthly share price index constructed by averaging the prices of common stock returns, then calculating the relative prices comparing to the average price in 2015 (provided by OECD database)
- Construct monthly return innovation:

$$MR_{i,t} = (P_{i,t} - P_{i,t-1}) / P_{i,t-1} * 100$$

$$RI_{i,t} = MR_{i,t} - MR_{i,t-1}$$

- Time window: March 1999 to April 2021, 266 sample months

Application: Risk Spillover Among Eurozone Countries
Data Description

- Summary Statistics:

Table 17: Summary Statistics of Stock Return Innovations (%)

	Mean	S.D.	Minimum	Maximum
Belgium	.0204	6.0182	-23.7896	28.2239
Germany	.0006	6.0780	-26.9865	21.5954
Spain	.0178	7.3668	-31.3490	30.7723
France	.0054	6.0733	-22.8660	23.5420
Italy	.0395	7.3879	-32.5638	28.9739
Netherland	.0295	5.5467	-23.1061	19.4025
Portugal	-.0034	9.0223	-26.9166	51.9512
Austria	.0073	5.4106	-24.6367	27.3145
Finland	.0157	5.6129	-23.7090	23.3261
Luxembourg	.0260	5.7835	-22.2640	27.0218
Ireland	.0181	6.637	-27.5134	31.5424

Application: Risk Spillover Among Eurozone Countries

Data Description

- Correlations:

Table 18: Sample Correlation Coefficients Between Stock Return Innovations

	BEL	DEU	ESP	FRA	ITA	NLD	PRT	AUT	FIN	LUX	IRL
BEL	1										
DEU	.82	1									
ESP	.71	.61	1								
FRA	.78	.67	.55	1							
ITA	.78	.69	.58	.70	1						
NLD	.77	.72	.64	.63	.64	1					
PRT	.66	.58	.43	.57	.52	.49	1				
AUT	.80	.72	.54	.74	.69	.67	.83	1			
FIN	.93	.87	.75	.79	.76	.78	.69	.84	1		
LUX	.92	.80	.71	.77	.79	.75	.73	.87	.94	1	
IRL	.87	.86	.70	.74	.70	.79	.61	.76	.91	.86	1

Application: Risk Spillover Among Eurozone Countries

Data Description

- Unit root, stationarity and ARCH tests: stationary with ARCH type heteroskedasticity

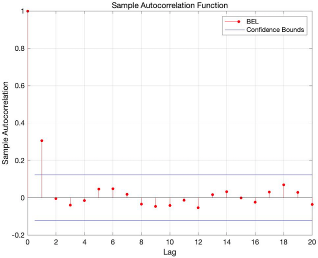
Table 19: ADF, KPSS and Engle's ARCH Test Results (Lag=12)

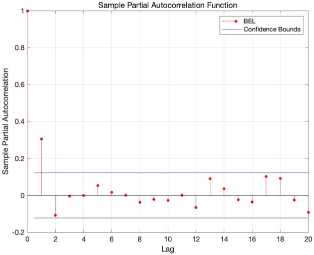
	BEL	DEU	ESP	FRA	ITA	NLD	RPT	AUT	FIN	LUX	IRL
ADF	-7.52	-7.39	-6.79	-6.88	-7.07	-7.88	-7.98	-7.12	-7.01	-7.35	-7.30
$p-$value	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00
KPSS	.0257	.0247	.0224	.0266	.0318	.0321	.0237	.0248	.0272	.0261	.0245
$p-$value[8]	>.10	>.10	>.10	>.10	>.10	>.10	>.10	>.10	>.10	>.10	>.10
Engles's	28.84	26.19	40.78	38.40	39.10	19.94	44.73	46.55	34.21	34.72	21.03
$p-$value	.00	.01	.00	.00	.00	.07	.00	.00	.00	.00	.15

Application: Risk Spillover Among Eurozone Countries

Data Description

- However, the AC and PAC functions of the return innovations are not regular, no clear evidence for autoregressive effects





- ESPARCH(1,1) specification:

$$RI_{i,t} = \sigma_{i,t}\varepsilon_{i,t}, \varepsilon_{i,t} \overset{iid}{\sim}(0,1)$$

$$\ln \sigma^2_{i,t} = \mu_i + \lambda \sum_{j=1}^{n} w_{ij,n} \ln RI^2_{j,t} + \gamma \ln RI^2_{i,t-1} + \rho \sum_{j=1}^{n} w_{ij,n} \ln RI^2_{j,t-1}$$

- Four different networks with row-normalized adjacent matrices :

1. $W_{adjacent}$: neighborhood defined by land adjacent relationship
2. $W_{distinv}$: inverse of distance among capital cites
3. $W_{dist2inv}$: inverse of square of distance among capital cites
4. W_{EV}: same weight for every other countries

Application: Risk Spillover Among Eurozone Countries
Empirical Specification and Results

- Pre-estimation normality test:

Table 20: J_{Norm} Test Results for Different Spatial Correlations

	J_{Norm} Statistic	p-Value
$W_{adjacent}$	.1581	.6909
$W_{distinv}$	17.7100	.0000
$W_{dist2inv}$	10.6971	.0011
W_{EU}	20.7423	.0000

- Major Specification Results:

Table 21: Major Specification for Different Spatial Correlations

	$W_{adjacent}$	$W_{distinv}$	$W_{dist2inv}$	W_{EU}
σ^2		4.1053***	4.2818***	4.0382***
		(.0351)	(.0375)	(.0346)
λ	.3005***	.5455***	.4634***	.5752***
	(.0353)	(.0399)	(.0457)	(.0353)
γ	.0808***	.0479***	.0565***	.0456***
	(.0198)	(.0185)	(.0194)	(.0175)
ρ	.1219***	.0925**	.0942**	.0750*
	(.0464)	(.0542)	(.0586)	(.0495)
quasi-LogLike	-6467.5	-6272.5	-6330.5	-6249.2
AIC	12963.0	12575.0	12691.0	12528.4
McFadden R^2	.3266	.3469	.3409	.3493
Efron R^2	.1357	.2659	.2343	.2777

Application: Risk Spillover Among Eurozone Countries

Empirical Specification and Results

1. W_{EU} is the best approximation of volatility spillovers in eurozone, which indicates that institutional links are more important economic links among the eurozone countries than geographical links (consistent with results in Blasques et al. (2016))
2. Both intra- and inter-temporal spillovers dominate the effect from own past history, which indicates risk transmission through network might be a more important pattern than time-series effect. For investors and policy makers, risk management should not only focus on local market.

Application: Risk Spillover Among Eurozone Countries

Comparison With Traditional Conditional Heteroskedasticity Models

1. single variate GARCH(1,1) in Bollerslev (1986):

$$\sigma_{i,t}^2 = \alpha_i + \beta_i \sigma_{i,t-1}^2 + \gamma_i RI_{i,t-1}^2$$

2. single variate EGARCH(1,1) in Nelson (1991):

$$\ln \sigma_{i,t}^2 = \kappa_i + \theta_i \ln \sigma_{i,t-1}^2 + \xi_i \varepsilon_{i,t-1}^2 + \iota_i \left(|\varepsilon_{i,t}| - E\,|\varepsilon_{i,t}| \right)$$

3. multivariate GARCH with constant correlation (CCC) in Bollerslev (1990):

$$E\left(y_t | \mathscr{F}_{t-1}\right) = 0$$

$$Var\left(y_t | \mathscr{F}_{t-1}\right) = \Omega_t$$

$$\sigma_{i,t}^2 = \omega_i + \alpha_i \sigma_{i,t-1}^2 + \beta_i y_{i,t-1}^2$$

$$\sigma_{ij,t} = \rho_{ij} \sigma_{i,t} \sigma_{j,t}$$

4. multivariate GARCH with dynamic conditional correlation (DCC) in Engle (2002) and Aielli (2013):
$Var\left(y_t|\mathscr{F}_{t-1}\right) = D_t^{1/2}R_tD_t^{1/2}$, where $R_t \equiv [\rho_{ij,t}]$ is the conditional correlation matrix and $D_t \equiv diag\left(h_{1,t},\cdots,h_{n,t}\right)$ is a diagonal matrix with the asset conditional variances as diagonal. R_t is the conditional covariance matrix of the standardized return innovations, $\epsilon_t \equiv \left[\epsilon_{1,t},\cdots,\epsilon_{n,t}\right]^{'}$, where $\epsilon_{i,t} \equiv y_{i,t}/\sqrt{h_{i,t}}$ elements

$$h_{i,t} = \omega_i + \alpha_i h_{i,t-1} + \beta_i y_{i,t-1}^2$$

$$R_t = Q_t^{*-1/2} Q_t Q_t^{*-1/2}$$

$$Q_t = \left(1-\lambda_1-\lambda_2\right)S + \lambda_1\epsilon_{t-1}^{'}\epsilon_{t-1} + \lambda_2 Q_{t-1}$$

$Q_t \equiv \left[q_{ij,t}\right]_{n\times n}$, $Q_t^* \equiv diag\left(q_{11,t},\cdots,q_{nn,t}\right)$, $S \equiv \left[s_{ij}\right]_{n\times n}$, and λ_1 and λ_2 are nonnegative scalars which satisfy $\lambda_1+\lambda_2<1$

Application: Risk Spillover Among Eurozone Countries

Comparison With Traditional Conditional Heteroskedasticity Models

- Our ESPARCH(1,1) model dominates the other specifications when comparing likelihood value and AIC criteria:

ESPARCH(1,1)	LogLike	AIC	Other Models	LogLike	AIC
$W_{adjacent}$	-6467.5	12963.0	GARCH(1,1)	-9353.6	18773.1
$W_{distinv}$	-6272.5	12575.0	EGARCH(1,1)	-9293.7	18675.3
$W_{dist2inv}$	-6330.5	12691.0	CCC	-7417.0	14900.0
W_{EU}	-6249.2	12528.4	DCC	-7304.7	14921.4

- With less parameters, our model can explain the conditional heteroskedasticity of eurozone stock return innovations better than existing models by introducing network risk spillovers