LM Tests for Heterogeneous Spatial Correlations with Application in Housing Market

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Introduction

Motivation

- Heterogeneity: individuals (or regions) $\Longrightarrow$ social network (or spillovers)

1. Gender difference in friendships : different interaction with classmates at school $\Longrightarrow$ heterogeneous peer effect on education outcome
2. Different city size: different level of externality received from neighborhood areas to local housing market

- Traditional Moran's I test is derived under homogeneous spatial correlations, which is not suitable for heterogeneous cases
- Single test is not enough for both existence and heterogeneity

Introduction

Empirical Interests

1. Matvos and Ostrivsky (2010): Mutual funds with some particular types tend to oppose other funds in corporate director elections
2. Yakusheva, Kapinos and Eisenberg (2014): Females are subject to peer influence in weight gain, with little evidence of peer effects for males in a natural experiment design for college student roommate assignment
3. Patacchini, Rainone and Zenou (2017): Peer effects on educational outcomes depend on the length of friendship

Introduction

Theoretical Literatures

1. Moran (1950), Cliff and Ord (1973): derive the Moran's I test statistic
2. Kelejian and Prucha (2001): derive the asymptotic property of Moran's I statistic for spatial autoregressive model (SAR)
3. Aquaro, Bailey and Pesaran (2020): spatial panel model with individual level heterogeneous coefficients

Heterogenous Coefficient Spatial Autoregressive Model

Basic Settings

- n individual spatial units in the economy located in a region $D_n \subset \mathbb{R}^d$, where $|D_n| = n$
- distance among individuals satisfy $d_{ij} \geq 1$ for any $i \neq j$
- K groups of individuals: K sub-regions $\left\{D_n^k\right\}_{k=1}^K$ inside D_n where K is constant and does not depend on n
- neighborhood relationship may not depend on D_n^k, for example, male and female students can be assigned into the same class

Heterogenous Coefficient Spatial Autoregressive Model

Model Formation and Interpretation

- DGP of HSAR model:

$$y_i = \sum_{k=1}^{K} \lambda_k h_{i,k} \left(\sum_{j=1}^{n} w_{ij} y_j \right) + x_i^{'} \beta + u_i$$

- $h_{i,k} = \begin{cases} 1 & i \in D_n^k \\ 0 & i \notin D_n^k \end{cases}$ and $u_i \overset{iid}{\sim} (0, \sigma^2)$
- w_{ij}: spatial weights, $w_{ij} \geq 0$ and $w_{ii} = 0$
- λ_k: neighborhood effect received by individual $i \in D_n^k$
- β: effects from other regressors

- Matrix Form:

$$y_n = \sum_{k=1}^{K} \lambda_k H_{n,k} W_n y_n + X_n \beta + u_n$$

- $W_n = (w_{ij})_{n\times n}$: spatial weighting matrix
- $H_{n,k} = diag\,(d_{1,k,}, \cdots, d_{n,k})$: diagonalized matrix of group dummy vectors, $\sum_{k=1}^{K} H_{n,k} = I_n$
- Without group heterogeneity, the model reduced to a standard SAR model: $y_n = \lambda W_n y_n + X_n \beta + u_n$

Heterogenous Coefficient Spatial Autoregressive Model

Economic Foundation

- Similar to SAR model, the HSAR can be regarded as a Nash equilibrium of a static complete information game with the following individual utility function:

$$u_i(y_i) = y_i\left(\lambda_k \sum_{j=1}^{n} w_{ij} y_j + x_i\beta + v_i\right) - \frac{y_i^2}{2}$$

- It can also be interpreted as a social interaction setting:

$$u_i(y_i) = \underbrace{y_i(x_i\beta + v_i)}_{private\ \ utility} \underbrace{- \frac{1}{2}\left(y_i - \lambda_k \sum_{j=1}^{n} w_{ij} y_j\right)^2}_{conformity\ \ effect\ \ with\ \ friends}$$

Heterogenous Coefficient Spatial Autoregressive Model

Likelihood Function

- With assuming $u_n \sim N\left(0, \sigma^2 I_n\right)$, the log-likelihood function is:

$$
\begin{aligned}
\ln L_n\left(\Lambda^{'}, \beta, \sigma^2\right) = & -\frac{n}{2}\ln(2\pi) - \frac{n}{2}\ln\sigma^2 + \ln|S_n(\Lambda)| \\
& - \frac{1}{2\sigma^2}\left(S_n(\Lambda)y_n - X_n\beta\right)'\left(S_n(\Lambda)y_n - X_n\beta\right)
\end{aligned}
$$

- $\Lambda = (\lambda_1, \cdots, \lambda_k)^{'}$ and $S_n(\Lambda) = I_n - \sum_{k=1}^{K} \lambda_k H_{n,k} W_n$
- To make sure $S_n(\Lambda)$ is invertible, a sufficient condition is $\max_k |\lambda_k| < \frac{1}{\|W_n\|_\infty}$
- Computationally cumbersome to maximize when sample size is large due to $\ln|S_n(\Lambda)|$ term

Test 1: Existence of Spatial Correlation

Test Statistic

- $H_0 : \lambda_k = 0$ *for* $\forall k = 1, \cdots, K$ vs. $H_1 : \exists k,\ \lambda_k \neq 0$
- Given MLE for linear regression model $\hat{\theta} = \left(0, \hat{\beta}', \hat{\sigma}^2\right)'$, we can obtain the FOC of constrained estimator:

$$g_{k,n}\left(\hat{\theta}\right) = \frac{\partial \ln L_n\left(\hat{\theta}\right)}{\partial \lambda_k} = \frac{1}{\hat{\sigma}^2}\left(y_n - X_n\hat{\beta}\right)' H_{n,k} W_n y_n = \frac{1}{\hat{\sigma}^2}\hat{u}' H_{n,k} W_n y_n$$

- Let $g_n\left(\hat{\theta}\right) = \frac{\partial \ln L_n\left(\hat{\theta}\right)}{\partial \theta} =$ $\left(g_{1,n}\left(\hat{\theta}\right), \cdots, g_{K,n}\left(\hat{\theta}\right), \underbrace{0, \cdots\cdots, 0}_{FOC\ \ of\ \ other\ \ parameters}\right)'$, then the LM test statistic is:

$$LM_1 = -g_n\left(\hat{\theta}\right)' \left(\frac{\partial^2 \ln L_n\left(\hat{\theta}\right)}{\partial \theta \partial \theta'}\right)^{-1} g_n\left(\hat{\theta}\right)$$

Test 1: Existence of Spatial Correlation

Asymptotic Distribution: Sketch of Proof

- Jointly asymptotic Normal$\Longleftrightarrow$ asymptotic Normal for any linear combinations
- Let $a = (a_1, \cdots, a_K)^{'}$ be an arbitrary vector of real numbers, we want to discuss:

$$f_n\left(a, \hat{\theta}\right) = \sum_{k=1}^{K} a_k g_n\left(\hat{\theta}\right) = \frac{1}{\hat{\sigma}^2} \hat{u}_n^{'} H_{a,n} W_n y_n$$

- With proper assumptions similar in Jenish and Prucha (2001) and Lee (2004), we have the following form:

$$\frac{1}{\sqrt{n}} f_n\left(a, \hat{\theta}\right) = \frac{1}{\hat{\sigma}^2 \sqrt{n}} \left(A_n^{'} u_n + u_n^{'} B_n u_n\right) + o_p(1)$$

- $H_{a,n}$, A_n and B_n are $n \times n$ matrices

Test 1: Existence of Spatial Correlation

Asymptotic Distribution: Sketch of Proof

- Two scenarios with different spatial weighting matrix:

1. $\frac{1}{\sqrt{n}} A_n^{'} u_n$ dominates $\frac{1}{\sqrt{n}} u_n^{'} B_n u_n$: apply Lyapunov CLT
2. $\frac{1}{\sqrt{n}} A_n^{'} u_n$ does not dominate: apply CLT for linear quadratic form in Jenish and Prucha (2001)

$\Longrightarrow$ Asymptotic Normality of $\frac{1}{\sqrt{n}} f_n \left(a, \hat{\theta} \right)$

$\Longrightarrow$ Jointly asymptotic Normality of $\frac{1}{\sqrt{n}} g_{k,n} \left(\hat{\theta} \right)$'s

Test 1: Existence of Spatial Correlation

Asymptotic Distribution: Sketch of Proof

- The asymptotic covariance matrix follows likelihood equality: $E_\theta\left(\frac{\partial^2 \ln L_n(\theta)}{\partial\theta\partial\theta'}\right) + E_\theta\left(\frac{\partial \ln L_n(\theta)}{\partial\theta}\frac{\partial \ln L_n(\theta)}{\partial\theta'}\right) = 0$
- Degree of freedom is K with the following regularity assumption:
- *For $\forall k = 1, \cdots, K$, we have $\lim_{n\to\infty} \frac{\left|D_n^k\right|}{n} = c_k$ where c_k is a non-zero positive constant and $\sum_{k=1}^{K} c_k = 1$, i.e. there exist a stationary distribution of types as $n \to \infty$ and the probability of each type would not shrink to zero.*
- Empirically, as long as you have large enough observations for each type, there is no problem
- Thus, we have $LM_1 \xrightarrow{d} \chi^2(K)$

Test 2: Heterogeneity among Spatial Correlation

Test Statistic

- $H_0 : \rho_1 = \cdots = \rho_K$ vs. $H_1 : \rho_i \neq \rho_j, \exists i \neq j$
- Given QMLE of SAR model $\bar{\theta} = \left(\bar{\Lambda}', \bar{\beta}', \bar{\sigma}^2\right)'$, we can obtain the FOC of constrained estimator:

$$h_{k,n}\left(\bar{\theta}\right) = \frac{\partial \ln L_n\left(\bar{\theta}\right)}{\partial \lambda_k} = \frac{1}{\bar{\sigma}^2}\bar{u}_n' H_{n,k} W_n y_n - tr\left[\left(I_n - \bar{\lambda} W_n\right)^{-1} H_{n,k} W_n\right]$$

- Let

$$h_n\left(\bar{\theta}\right) = \frac{\partial \ln L_n\left(\bar{\theta}\right)}{\partial \theta} = \left(h_{1,n}\left(\bar{\theta}\right), \cdots, h_{K,n}\left(\bar{\theta}\right), \underbrace{0, \cdots\cdots, 0}_{FOC\ \ of\ \ other\ \ parameters}\right)',$$

then the LM statistic is

$$LM_2 = -h_n\left(\bar{\theta}\right)' \left(\frac{\partial^2 \ln L_n\left(\bar{\theta}\right)}{\partial \theta \partial \theta'}\right)^{-1} h_n\left(\bar{\theta}\right)$$

Test 2: Heterogeneity among Spatial Correlation

Asymptotic Distribution: Sketch of Proof

- Similar to $LM1$, we need to prove the asymptotic Normality of the linear combinations of scores:

$$\xi_n\left(a,\bar{\theta}\right) = \frac{1}{\bar{\sigma}^2}\bar{u}_n' H_{a,n} W_n y_n - tr\left[\left(I_n - \bar{\lambda} W_n\right)^{-1} H_{a,n} W_n\right]$$

- The first term is similar to discussion for $LM1$, with slightly complicated discussions
- With regularity assumptions on W_n, $\frac{1}{\sqrt{n}} tr\left[\left(I_n - \bar{\lambda} W_n\right)^{-1} H_{a,n} W_n\right] = o_p(1)$

Test 2: Heterogeneity among Spatial Correlation

- Asymptotic Distribution: Sketch of Proof
- With the same assumption, the degree of freedom of $LM2$ is $(K-1)$ since:

$$\begin{aligned}
\sum_{k=1}^{K} h_{k,n}\left(\bar{\theta}\right) &= \sum_{k=1}^{K}\left\{\frac{1}{\bar{\sigma}^2}\bar{u}_n' H_{n,k} W_n y_n - tr\left[\left(I_n - \bar{\lambda} W_n\right)^{-1} H_{n,k} W_n\right]\right\} \\
&= \frac{1}{\bar{\sigma}^2}\bar{u}_n' W_n y_n - tr\left[\left(I_n - \bar{\lambda} W_n\right)^{-1} W_n\right] \\
&= 0
\end{aligned}$$

- Thus, we have $LM2 \xrightarrow{d} \chi^2(K-1)$

Monte Carlo Simulations

Basic Settings

- Spatial weighting matrix is constructed by the following way:

1. Generate two random vectors of coordinates as the geographic location for each observation;
2. Find l nearest neighbors for each observation according to their spatial distances and denote the corresponding $w_{ij} = 1$, otherwise $w_{ij} = 0$;
3. Row-normalize W_n.

- 1000 times replications for each round
- External regressor: x_1 intercept, $x_2 \overset{iid}{\sim} N(0, 1)$

Monte Carlo Simulations

Performance of $LM1$: Test Size

- In simulations for $LM1$, we have two groups with 4:1 ratio of individuals

Table 1: Test Size of LM_1 ($\chi^2_{0.95}(2) = 5.9915$)

n	neighbors	residuals	$(\beta', \sigma^2) = [(1,1), 4]$	$(\beta', \sigma^2) = [(2,-5), 1]$
100	$l=5$	$N(0,\sigma^2)$	0.068	0.071
		$\sigma[\Gamma(2.25,2)-4.5]$	0.072	0.073
		$\sigma U[-\sqrt{3},\sqrt{3}]$	0.067	0.061
	$l=10$	$N(0,\sigma^2)$	0.078	0.077
		$\sigma[\Gamma(2.25,2)-4.5]$	0.057	0.071
		$\sigma U[-\sqrt{3},\sqrt{3}]$	0.073	0.066
200	$l=5$	$N(0,\sigma^2)$	0.055	0.067
		$\sigma[\Gamma(2.25,2)-4.5]$	0.074	0.054
		$\sigma U[-\sqrt{3},\sqrt{3}]$	0.057	0.058
	$l=10$	$N(0,\sigma^2)$	0.058	0.064
		$\sigma[\Gamma(2.25,2)-4.5]$	0.054	0.059
		$\sigma U[-\sqrt{3},\sqrt{3}]$	0.067	0.054
400	$l=5$	$N(0,\sigma^2)$	0.048	0.049
		$\sigma[\Gamma(2.25,2)-4.5]$	0.050	0.048
		$\sigma U[-\sqrt{3},\sqrt{3}]$	0.056	0.048
	$l=10$	$N(0,\sigma^2)$	0.052	0.054
		$\sigma[\Gamma(2.25,2)-4.5]$	0.053	0.062
		$\sigma U[-\sqrt{3},\sqrt{3}]$	0.051	0.057

Monte Carlo Simulations

Performance of $LM1$: Test Power

- Compare to small power of Moran's I in some situations, the test power of $LM1$ is far better and converge to 1 as sample size increases

n	neighbors	residuals	$\left(\lambda_1, \lambda_2, \beta', \sigma^2\right) = [0, 0.4, (2, -5), 1]$ Moran's I Statistic	$LM1$ Statistic
100	$l=5$	$N\left(0, \sigma^2\right)$	0.055	0.933
		$\sigma\left[\Gamma(2.25, 2) - 4.5\right]$	0.091	0.437
		$\sigma U\left[-\sqrt{3}, \sqrt{3}\right]$	0.103	0.996
	$l=10$	$N\left(0, \sigma^2\right)$	0.085	0.763
		$\sigma\left[\Gamma(2.25, 2) - 4.5\right]$	0.083	0.395
		$\sigma U\left[-\sqrt{3}, \sqrt{3}\right]$	0.059	0.997
200	$l=5$	$N\left(0, \sigma^2\right)$	0.177	1
		$\sigma\left[\Gamma(2.25, 2) - 4.5\right]$	0.148	0.776
		$\sigma U\left[-\sqrt{3}, \sqrt{3}\right]$	0.213	1
	$l=10$	$N\left(0, \sigma^2\right)$	0.132	1
		$\sigma\left[\Gamma(2.25, 2) - 4.5\right]$	0.102	0.547
		$\sigma U\left[-\sqrt{3}, \sqrt{3}\right]$	0.111	1
400	$l=5$	$N\left(0, \sigma^2\right)$	0.176	1
		$\sigma\left[\Gamma(2.25, 2) - 4.5\right]$	0.189	0.985
		$\sigma U\left[-\sqrt{3}, \sqrt{3}\right]$	0.237	1
	$l=10$	$N\left(0, \sigma^2\right)$	0.193	1
		$\sigma\left[\Gamma(2.25, 2) - 4.5\right]$	0.146	0.726
		$\sigma U\left[-\sqrt{3}, \sqrt{3}\right]$	0.143	1

Monte Carlo Simulations

Performance of *LM*2: Test Size

- In simulations for *LM*2, we have three groups with 3:5:2 ratio of individuals

Table 5: Test Size of LM_2 ($\chi^2_{0.95}(2) = 5.9915$)

n	neighbors	residuals	$(\lambda, \beta', \sigma^2) = [0.5, (1,1), 4]$	$(\lambda, \beta', \sigma^2) = [-0.4, (2,-5), 1]$
100	$l = 5$	$N(0, \sigma^2)$	0.078	0.077
		$\sigma[\Gamma(2.25, 2) - 4.5]$	0.112	0.054
		$\sigma U[-\sqrt{3}, \sqrt{3}]$	0.067	0.080
	$l = 10$	$N(0, \sigma^2)$	0.097	0.069
		$\sigma[\Gamma(2.25, 2) - 4.5]$	0.093	0.058
		$\sigma U[-\sqrt{3}, \sqrt{3}]$	0.070	0.071
200	$l = 5$	$N(0, \sigma^2)$	0.062	0.065
		$\sigma[\Gamma(2.25, 2) - 4.5]$	0.095	0.067
		$\sigma U[-\sqrt{3}, \sqrt{3}]$	0.600	0.057
	$l = 10$	$N(0, \sigma^2)$	0.067	0.058
		$\sigma[\Gamma(2.25, 2) - 4.5]$	0.085	0.057
		$\sigma U[-\sqrt{3}, \sqrt{3}]$	0.045	0.059
400	$l = 5$	$N(0, \sigma^2)$	0.054	0.047
		$\sigma[\Gamma(2.25, 2) - 4.5]$	0.082	0.050
		$\sigma U[-\sqrt{3}, \sqrt{3}]$	0.056	0.049
	$l = 10$	$N(0, \sigma^2)$	0.059	0.054
		$\sigma[\Gamma(2.25, 2) - 4.5]$	0.077	0.049
		$\sigma U[-\sqrt{3}, \sqrt{3}]$	0.048	0.048

Monte Carlo Simulations

Performance of $LM2$: Test Power

Table 6: Test Power of LM_2 ($\chi^2_{0.95}(2) = 5.9915$)

n	neighbors	residuals	$\left(\lambda_1, \lambda_2, \lambda_3, \beta', \sigma^2\right) = [0.5, -0.2, 0.7, (1, 1), 4]$	$\left(\lambda_1, \lambda_2, \lambda_3, \beta', \sigma^2\right) = [0, 0.4, 0.1, (2, -5), 1]$
100	$l = 5$	$N\left(0, \sigma^2\right)$	0.790	0.921
		$\sigma\left[\Gamma(2.25, 2) - 4.5\right]$	0.496	0.218
		$\sigma U\left[-\sqrt{3}, \sqrt{3}\right]$	0.827	0.862
	$l = 10$	$N\left(0, \sigma^2\right)$	0.834	0.673
		$\sigma\left[\Gamma(2.25, 2) - 4.5\right]$	0.315	0.186
		$\sigma U\left[-\sqrt{3}, \sqrt{3}\right]$	0.603	0.882
200	$l = 5$	$N\left(0, \sigma^2\right)$	0.971	0.989
		$\sigma\left[\Gamma(2.25, 2) - 4.5\right]$	0.775	0.453
		$\sigma U\left[-\sqrt{3}, \sqrt{3}\right]$	0.982	0.997
	$l = 10$	$N\left(0, \sigma^2\right)$	0.946	0.931
		$\sigma\left[\Gamma(2.25, 2) - 4.5\right]$	0.531	0.343
		$\sigma U\left[-\sqrt{3}, \sqrt{3}\right]$	0.967	0.984
400	$l = 5$	$N\left(0, \sigma^2\right)$	1	1
		$\sigma\left[\Gamma(2.25, 2) - 4.5\right]$	0.963	0.715
		$\sigma U\left[-\sqrt{3}, \sqrt{3}\right]$	1	1
	$l = 10$	$N\left(0, \sigma^2\right)$	0.999	0.998
		$\sigma\left[\Gamma(2.25, 2) - 4.5\right]$	0.811	0.580
		$\sigma U\left[-\sqrt{3}, \sqrt{3}\right]$	1	1

Application: City Size and Housing Market

Short-run Effect of Size Heterogeneity

- Data: annual housing price index change rate from 2006 to 2014, 240 counties in Northeastern US
- Cross-sectional regression for each year (reduce long-run reverse effect)
- Large city areas: By using Census 2010 population size, the largest 10 MSAs with more than 1 million residents and their encompassing CSA counties are classified as large city areas
- The spatial weighting matrix we use is the row-normalized county adjacent matrix

Application: City Size and Housing Market

Alternative Model Specifications

1. Linear Regression:

$$\Delta HPI\%_{i,t} = \beta_0 + \beta_1 \Delta realGDP_{i,t} + \beta_2 Large_i + State_i + \varepsilon_i$$

2. SAR model:

$$\Delta HPI\%_{i,t} = \beta_0 + \rho \sum_{j=1}^{n} w_{ij} \Delta HPI\%_{j,t} + \beta_1 Large_i + \beta_2 \Delta realGDP_{i,t} + \beta_3 \sum_{j=1}^{n} w_{ij} \Delta realGDP_{j,t} + State_i + \varepsilon_i$$

3. HSAR model:

$$\Delta HPI\%_{i,t} = \rho_L Large_i \sum_{j=1}^{n} w_{ij} \Delta HPI\%_{j,t} + \rho_S (1 - Large_i) \sum_{j=1}^{n} w_{ij} \Delta HPI\%_{j,t} + \beta_0 + \beta_1 Large_i + \beta_2 \Delta realGDP_{i,t} + \beta_3 \sum_{j=1}^{n} w_{ij} \Delta realGDP_{j,t} + State_i + \varepsilon_i$$

Application: City Size and Housing Market

Pre-estimation Test Results

- Moran's I and $LM1$ indicates a strong spatial correlation among the $\Delta HPI_{i,t}$ despite 2013
- $LM2$ indicates a time-varying heterogeneity of the spatial correlations, which is stronger in 2006, 2007 and 2014 when large city areas have positive annual housing price growth on average

Table 12: Test Results of Moran's I, $LM1$ and $LM2$

		2006	2007	2008	2009	2010	2011	2012	2013	2014
Moran	Statistic	8.39	2.06	6.09	7.99	5.74	7.86	10.07	1.37	2.08
	p-value	.00	.04	.00	.00	.00	.00	.00	.17	.04
$LM1$	Statistic	87.83	9.56	41.75	65.92	29.13	69.44	119.91	2.60	6.70
	p-value	.00	.01	.00	.00	.00	.00	.00	.27	.04
$LM2$	Statistic	8.07	3.73	2.05	.62	.10	2.61	1.14	1.33	4.72
	p-value	.00	.05	.15	.43	.75	.11	.29	.25	.03

Application: City Size and Housing Market

Results from HSAR Specification

Table 15: Results of Model 3 (HSAR)

	2006	2007	2008	2009	2010	2011	2012	2013	2014
ρ_L	.67***	.36***	.50***	.50***	.35***	.34***	.53***	.21	.33**
	(.08)	(.12)	(.09)	(.08)	(.11)	(.10)	(.10)	(.15)	(.14)
ρ_S	.31***	.05	.33***	.41***	.31***	.55***	.66***	-.02	-.08
	(.10)	(.12)	(.09)	(.08)	(.10)	(.09)	(.08)	(.13)	(.12)
β_0	5.50***	2.08***	.60	-1.77**	-2.23***	-1.15**	-.43	-.80	-1.3**
	(1.09)	(.67)	(.56)	(.69)	(.66)	(.53)	(.47)	(.50)	(.63)
β_1	-3.35***	-2.72***	-1.01***	-.50	.73	-.71*	.22	.56*	1.92***
	(.99)	(.53)	(.38)	(.57)	(.50)	(.40)	(.32)	(.32)	(.50)
β_2	.10***	-.01	-.03	.02	.07**	.14***	.01	.05	-.02
	(.04)	(.04)	(.04)	(.04)	(.03)	(.04)	(.03)	(.04)	(.04)
β_3	.11	.05	.13	.13	.19***	.16*	.00	.02	.06
	(.09)	(.08)	(.09)	(.08)	(.07)	(.09)	(.07)	(.07)	(.07)
R^2	.84	.59	.61	.82	.75	.67	.53	.17	.44

Application: City Size and Housing Market

Major Results from HSAR Specification

- Time Varying Heterogeneity of City Size:

1. β_1: from significantly negative to significantly positive from 2006 to 2014
2. $\rho_L - \rho_S$: large cities received more spill-over effects when their housing market is growing in 2006, 2007 and 2014, but the difference disappear during recession

- Post-estimation $t-$statistics are consistent with pre-estimation $LM2$ test statistics:

Table 16: Post Estimation t-test for $H_0 : \rho_L = \rho_S$

		2006	2007	2008	2009	2010	2011	2012	2013	2014
t-statistic	Statistic	2.81	1.93	1.42	.78	.30	-1.60	-1.05	1.14	2.13
	p-value	.01	.05	.16	.44	.76	.11	.29	.25	.03

Application: City Size and Housing Market

Why city size matters?

- Credit Cycle and Uneven Income Distribution Across Regions

1. Mian and Sufi (2009,2015), Adelino, Schoar and Severino (2015): Low income buyers contributes increasing share of delinquencies from 2003 to 2008, including lower-half of middle class
2. Baum-Snow and Pavan (2013): Inequality among wages is strong positively correlated with city size
3. JCHS of Harvard University: higher housing price to income ratio in large cities

- The housing market in large cities are more sensitive to credit cycles due to more lower income borrowers and higher leverage rate

Application: City Size and Housing Market

Financial Crisis & Geographical Income Inequality

- Higher degree of Inequality:

1. Credit Expansion: Housing Demand$\uparrow \Longrightarrow$Housing Price$\uparrow$ $\Rightarrow$ Leverage Rate$\uparrow$ (Systematic Risk$\uparrow$)
2. Credit Crunch: Delinquency$\uparrow \Rightarrow$ Housing Demand $\downarrow$ & Supply $\uparrow \Rightarrow$ Housing Price$\downarrow$

- The result provides indirect evidence for Kumhof, Ranciere and Winant (2015), that financial crisis can be caused by dynamic of income distribution, with considering the variations across space.